Chapter 7

Space and Time Tradeoffs

Space-for-time tradeoffs

Two varieties of space-for-time algorithms:

- ***input enhancement*** **— preprocess the input (or its part) to store some info to be used later in solving the problem**
  - **counting sorts**
  - **string searching algorithms**

- ***prestructuring*** **— preprocess the input to make accessing its elements easier**
  - **hashing**
  - **indexing schemes (e.g., B-trees)**

Count Sorting

Array *A*[0..5]		62	31	84	96	19	47
Initially	*Count* []	0	0	0	0	0	0
After pass *i* = 0	*Count* []	3	0	1	1	0	0
After pass *i* = 1	*Count* []		1	2	2	0	1
After pass *i* = 2	*Count* []			4	3	0	1
After pass *i* = 3	*Count* []				5	0	1
After pass *i* = 4	*Count* []					0	2
Final state	*Count* []	3	1	4	5	0	2
Array *S*[0..5]		19	31	47	62	84	96

FIGURE 7.1 Example of sorting by comparison counting

ALGORITHM *ComparisonCountingSort*$(A[0..n-1])$

```
//Sorts an array by comparison counting
//Input: An array A[0..n − 1] of orderable elements
//Output: Array S[0..n − 1] of A's elements sorted in nondecreasing order
for i ← 0 to n − 1 do Count[i] ← 0
for i ← 0 to n − 2 do
    for j ← i + 1 to n − 1 do
        if A[i] < A[j]
            Count[j] ← Count[j] + 1
        else Count[i] ← Count[i] + 1
for i ← 0 to n − 1 do S[Count[i]] ← A[i]
return S
```

	D[0..2]			S[0..5]					
$A[5] = 12$	1	**4**	6				12		
$A[4] = 12$	1	**3**	6			12			
$A[3] = 13$	1	2	**6**						13
$A[2] = 12$	1	**2**	5		12				
$A[1] = 11$	**1**	1	5	11					
$A[0] = 13$	0	1	**5**					13	

FIGURE 7.2 Example of sorting by distribution counting. The distribution values being decremented are shown in bold.

ALGORITHM *DistributionCounting*$(A[0..n-1], l, u)$

```
//Sorts an array of integers from a limited range by distribution counting
//Input: An array A[0..n − 1] of integers between l and u (l ≤ u)
//Output: Array S[0..n − 1] of A's elements sorted in nondecreasing order
for j ← 0 to u − l do D[j] ← 0                          //initialize frequencies
for i ← 0 to n − 1 do D[A[i] − l] ← D[A[i] − l] + 1     //compute frequencies
for j ← 1 to u − l do D[j] ← D[j − 1] + D[j]            //reuse for distribution
for i ← n − 1 downto 0 do
    j ← A[i] − l
    S[D[j] − 1] ← A[i]
    D[j] ← D[j] − 1
return S
```

Review: String searching by brute force

pattern: a string of *m* characters to search for

text: a (long) string of *n* characters to search in

Brute force algorithm

Step 1 Align pattern at beginning of text

Step 2 Moving from left to right, compare each character of pattern to the corresponding character in text until either all characters are found to match (successful search) or a mismatch is detected

Step 3 While a mismatch is detected and the text is not yet exhausted, realign pattern one position to the right and repeat Step 2

String searching by preprocessing

Several string searching algorithms are based on the input enhancement idea of preprocessing the pattern

- Knuth-Morris-Pratt (KMP) algorithm preprocesses pattern left to right to get useful information for later searching

- Boyer -Moore algorithm preprocesses pattern right to left and store information into two tables

- Horspool's algorithm simplifies the Boyer-Moore algorithm by using just one table

Horspool's Algorithm

A simplified version of Boyer-Moore algorithm:

- **preprocesses pattern to generate a shift table that determines how much to shift the pattern when a mismatch occurs**

- **always makes a shift based on the text's character *c* aligned with the last character in the pattern according to the shift table's entry for *c***

How far to shift?

Look at first (rightmost) character in text that was compared:

- The character is not in the pattern

  `......c.........................` (c not in pattern)

  `BAOBAB`

- The character is in the pattern (but not the rightmost)

  `......O.........................` (O occurs once in pattern)

  `BAOBAB`

  `......A.........................` (A occurs twice in pattern)

  `BAOBAB`

- The rightmost characters do match

  `......B.........................`

  `BAOBAB`

Shift table

- Shift sizes can be precomputed by the formula

  distance from *c*'s rightmost occurrence in pattern among its first *m*-1 characters to its right end

  $t(c)$ =

  pattern's length *m*, otherwise

  by scanning pattern before search begins and stored in a table called *shift table*

- Shift table is indexed by text and pattern alphabet
  Eg, for **BAOBAB** :

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z
1	2	6	6	6	6	6	6	6	6	6	6	6	6	3	6	6	6	6	6	6	6	6	6	6	6

Example of Horspool's alg. application

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
1	2	6	6	6	6	6	6	6	6	6	6	6	6	3	6	6	6	6	6	6	6	6	6	6	6	6

```
BARD LOVED BANANAS
BAOBAB
   BAOBAB
    BAOBAB
       BAOBAB
```

(unsuccessful search)

ALGORITHM *ShiftTable*($P[0..m-1]$)

//Fills the shift table used by Horspool's and Boyer-Moore algorithms
//Input: Pattern $P[0..m-1]$ and an alphabet of possible characters
//Output: $Table[0..size-1]$ indexed by the alphabet's characters and
// filled with shift sizes computed by formula (7.1)
initialize all the elements of *Table* with m
for $j \leftarrow 0$ **to** $m-2$ **do** $Table[P[j]] \leftarrow m-1-j$
return *Table*

ALGORITHM *HorspoolMatching*$(P[0..m-1], T[0..n-1])$

```
//Implements Horspool's algorithm for string matching
//Input: Pattern P[0..m − 1] and text T[0..n − 1]
//Output: The index of the left end of the first matching substring
//        or −1 if there are no matches
ShiftTable(P[0..m − 1])      //generate Table of shifts
i ← m − 1                    //position of the pattern's right end
while i ≤ n − 1 do
    k ← 0                    //number of matched characters
    while k ≤ m − 1 and P[m − 1 − k] = T[i − k] do
        k ← k + 1
    if k = m
        return i − m + 1
    else  i ← i+Table[T[i]]
return −1
```

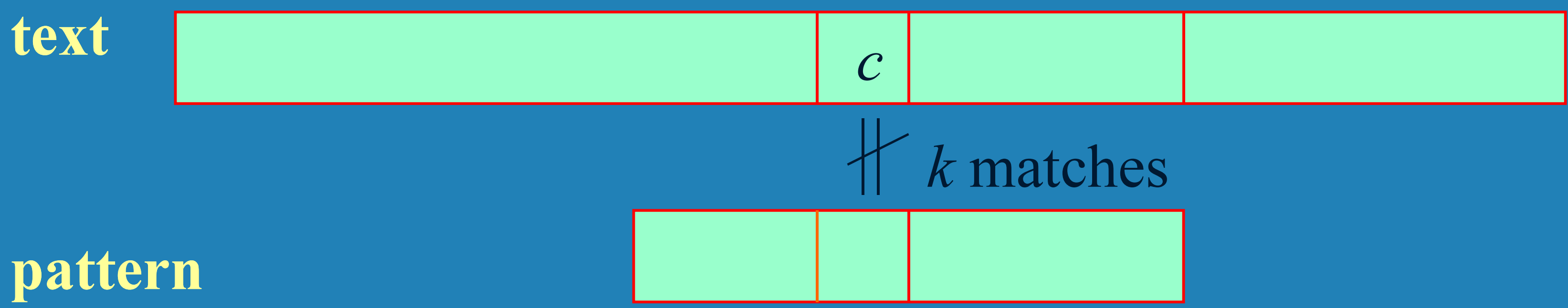
Boyer-Moore algorithm

Based on same two ideas:

- **comparing pattern characters to text from right to left**
- **precomputing shift sizes in two tables**
  - ***bad-symbol table* indicates how much to shift based on text's character causing a mismatch**
  - ***good-suffix table* indicates how much to shift based on matched part (suffix) of the pattern**

Bad-symbol shift in Boyer-Moore algorithm

- **If the rightmost character of the pattern doesn't match, BM algorithm acts as Horspool's**
- **If the rightmost character of the pattern does match, BM compares preceding characters right to left until either all pattern's characters match or a mismatch on text's character *c* is encountered after $k > 0$ matches**



bad-symbol shift $d_1 = \max\{t_1(c) - k, 1\}$

Good-suffix shift in Boyer-Moore algorithm

- Good-suffix shift d_2 is applied after $0 < k < m$ last characters were matched
- $d_2(k)$ = the distance between matched suffix of size k and its rightmost occurrence in the pattern that is not preceded by the same character as the suffix

  Example: `CABABA` $d_2(1) = 4$

- If there is no such occurrence, match the longest part of the k-character suffix with corresponding prefix;
  if there are no such suffix-prefix matches, $d_2(k) = m$

  Example: `WOWWOW` $d_2(2) = 5$, $d_2(3) = 3$, $d_2(4) = 3$, $d_2(5) = 3$

Boyer-Moore Algorithm

After matching successfully $0 < k < m$ characters, the algorithm shifts the pattern right by

$$d = \max\{d_1, d_2\}$$

where $d_1 = \max\{t_1(c) - k, 1\}$ is bad-symbol shift

$d_2(k)$ is good-suffix shift

Example: Find pattern `AT_THAT` in

`WHICH_FINALLY_HALTS. _ _AT_THAT`

Boyer-Moore Algorithm (cont.)

Step 1 Fill in the bad-symbol shift table

Step 2 Fill in the good-suffix shift table

Step 3 Align the pattern against the beginning of the text

Step 4 Repeat until a matching substring is found or text ends:

Compare the corresponding characters right to left.

If no characters match, retrieve entry $t_1(c)$ from the bad-symbol table for the text's character c causing the mismatch and shift the pattern to the right by $t_1(c)$.

If $0 < k < m$ characters are matched, retrieve entry $t_1(c)$ from the bad-symbol table for the text's character c causing the mismatch and entry $d_2(k)$ from the good-suffix table and shift the pattern to the right by

$$d = \max \{d_1, d_2\}$$

where $d_1 = \max\{t_1(c) - k, 1\}$.

Example of Boyer-Moore alg. application

A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	_
1	2	6	6	6	6	6	6	6	6	6	6	6	6	3	6	6	6	6	6	6	6	6	6	6	6	6

```
B E S S _ K N E W _ A B O U T _ B A O B A B S
B A O B A B
            B A O B A B
                              B A O B A B
                                          B A O B A B (success)
```

$d_1 = t_1(\text{K}) = 6$

$d_1 = t_1(\_)-2 = 4$

$\underline{d_2(2) = 5}$

$\underline{d_1 = t_1(\_)-1 = 5}$

$d_2(1) = 2$

k	pattern	d_2
1	BAO**B**A**B**	2
2	**B**AOB**AB**	5
3	**B**AO**BAB**	5
4	**B**A**OBAB**	5
5	**BAOBAB**	5

Hashing

- A very efficient method for implementing a *dictionary*, i.e., a set with the operations:
  - find
  - insert
  - delete

- Based on representation-change and space-for-time tradeoff ideas

- Important applications:
  - symbol tables
  - databases (*extendible hashing*)

Hash tables and hash functions

The idea of *hashing* is to map keys of a given file of size n into a table of size m, called the *hash table*, by using a predefined function, called the *hash function*,

$$h: K \rightarrow \text{ location (cell) in the hash table}$$

Example: student records, key = SSN. Hash function:
$h(K) = K \bmod m$ where m is some integer (typically, prime)
If $m = 1000$, where is record with SSN= 314159265 stored?

Generally, a hash function should:

- be easy to compute
- distribute keys about evenly throughout the hash table

Collisions

If $h(K_1) = h(K_2)$, there is a *collision*

- Good hash functions result in fewer collisions but some collisions should be expected (*birthday paradox*)

- Two principal hashing schemes handle collisions differently:
  - *Open hashing*
    – each cell is a header of linked list of all keys hashed to it
  - *Closed hashing*
    - one key per cell
    - in case of collision, finds another cell by
      - *linear probing:* use next free bucket
      - *double hashing:* use second hash function to compute increment

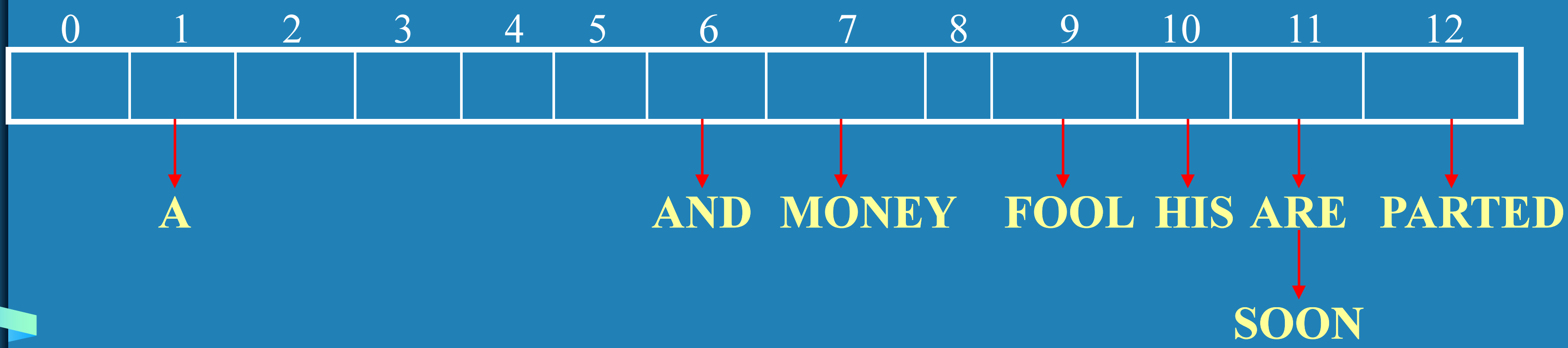
Open hashing (Separate chaining)

Keys are stored in linked lists <u>outside</u> a hash table whose elements serve as the lists' headers.

Example: A, FOOL, AND, HIS, MONEY, ARE, SOON, PARTED

$h(K)$ = sum of K 's letters' positions in the alphabet MOD 13

Key	A	FOOL	AND	HIS	MONEY	ARE	SOON	PARTED
$h(K)$	1	9	6	10	7	11	11	12



Search for KID

Open hashing (cont.)

- If hash function distributes keys uniformly, average length of linked list will be $\alpha = n/m$. This ratio is called *load factor*.

- Average number of probes in successful, S, and unsuccessful searches, U:

$$S \approx 1+\alpha/2, \qquad U = \alpha$$

- Load α is typically kept small (ideally, about 1)

- Open hashing still works if $n > m$

Closed hashing (Open addressing)

Keys are stored <u>inside</u> a hash table.

Key	A	FOOL	AND	HIS	MONEY	ARE	SOON	PARTED
$h(K)$	1	9	6	10	7	11	11	12

0	1	2	3	4	5	6	7	8	9	10	11	12
	A											
	A								FOOL			
	A					AND			FOOL			
	A					AND			FOOL	HIS		
	A					AND	MONEY		FOOL	HIS		
	A					AND	MONEY		FOOL	HIS	ARE	
	A					AND	MONEY		FOOL	HIS	ARE	SOON
PARTED	A					AND	MONEY		FOOL	HIS	ARE	SOON

Closed hashing (cont.)

- Does not work if $n > m$
- Avoids pointers
- Deletions are *not* straightforward
- Number of probes to find/insert/delete a key depends on load factor $\alpha = n/m$ (hash table density) and collision resolution strategy. For linear probing:

  $S = (½)\ (1+ 1/(1- \alpha))$ and $U = (½)\ (1+ 1/(1- \alpha)^2)$
- As the table gets filled (α approaches 1), number of probes in linear probing increases dramatically:

α	$\frac{1}{2}(1+\frac{1}{1-\alpha})$	$\frac{1}{2}(1+\frac{1}{(1-\alpha)^2})$
50%	1.5	2.5
75%	2.5	8.5
90%	5.5	50.5

A B-tree of order *m* (the maximum number of children for each node) is a tree which satisfies the following properties:

- Every node has at most *m* children.
- Every node (except root and leaves) has at least *m*/2 children.
- The root has at least two children if it is not a leaf node.
- All leaves appear in the same level, and carry information.
- A non-leaf node with *k* children contains *k*–1 keys.

B Tree

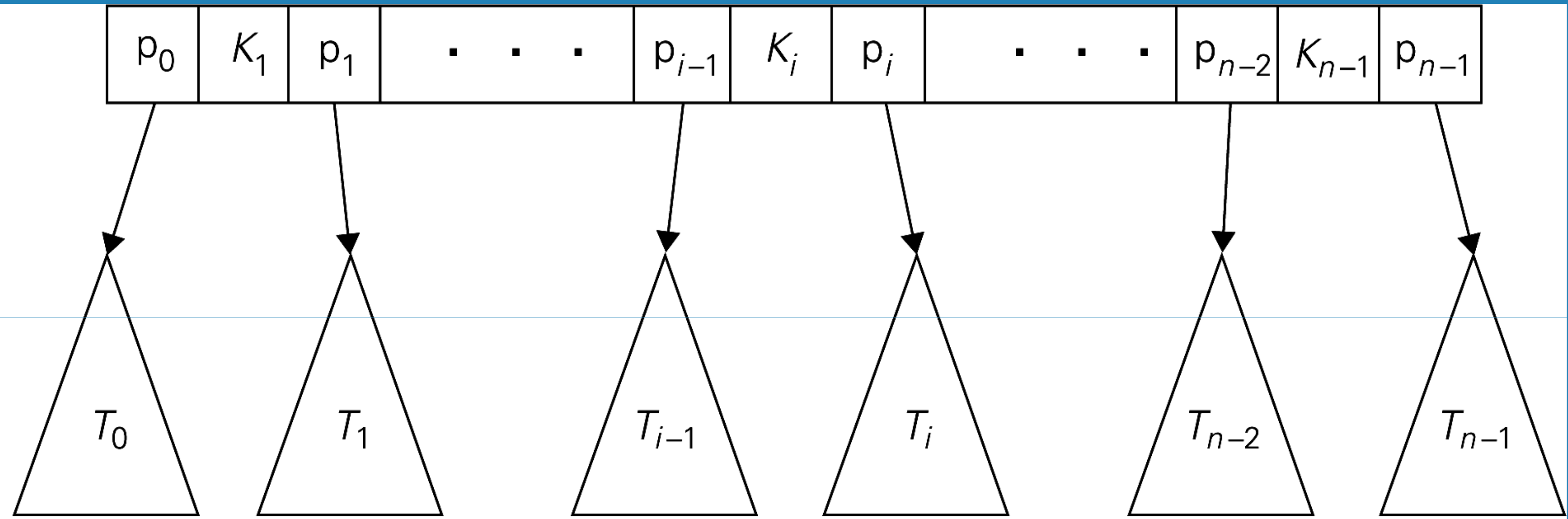


FIGURE 7.7 Parental node of a B-tree

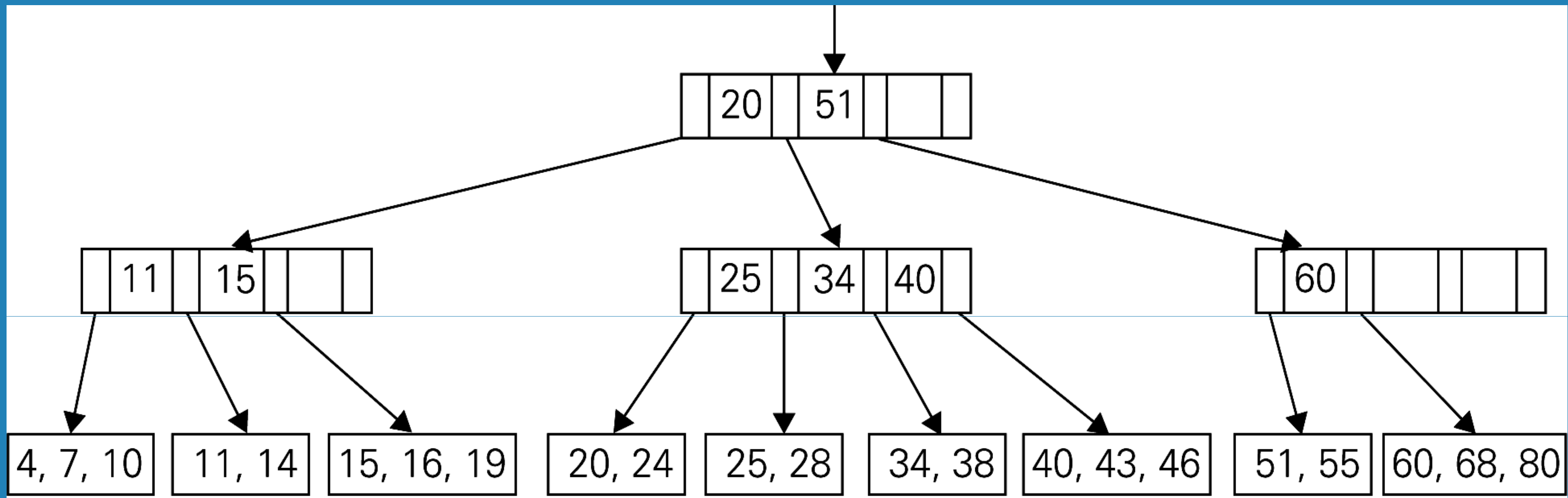


FIGURE 7.8 Example of a B-tree of order 4

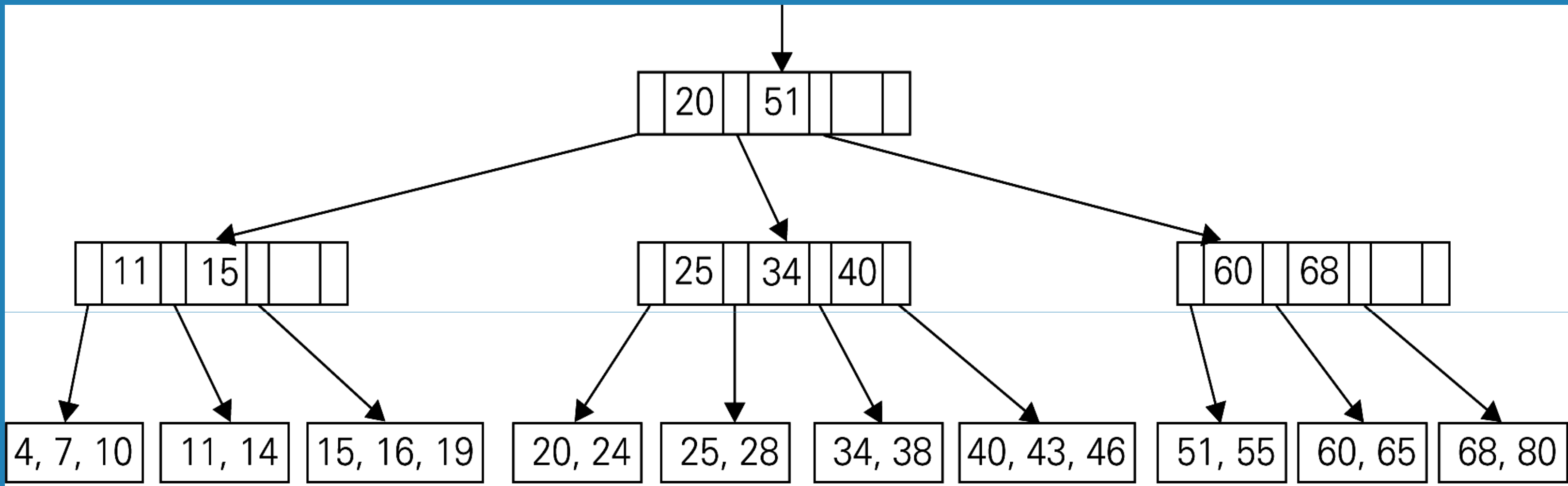


FIGURE 7.9 B-tree obtained after inserting 65 into the B-tree in Figure 7.8

Excersie

Section 7.1 4, 5, 6

Section 7.2 4, 9

Section 7.3 5, 8

Section 7.4 2,5